Leontief Input-Output Analysis

Finite Math

12 April 2017

Quiz

Which is better?

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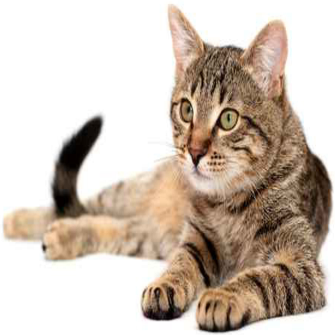

Two-Industry Model

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To simplify the ideas, we will assume we are in an economy with only two industries: coal and steel. In this economy, to produce \$1 worth of coal requires an input of \$0.10 from the coal sector and \$0.20 from the steel sector; and to produce \$1 worth of steel requires an input of \$0.20 from the coal sector and \$0.40 from the steel sector. The final demand (the demand from all other users of coal and steel) is \$20 billion for coal and \$10 billion for steel. What we would like to know is how much total coal and steel needs to be produced to meet this final demand.

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This only leaves $16 billion of coal and $2 billion of steel left over to meet that final demand, well below the required amounts.

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$$
\begin{array}{rcccl}
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y & = & 0.2x + 0.4y & + & 10
\end{array}
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which we rewrite in matrix form as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 20 \\ 10 \end{bmatrix}$$

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D is called the *final demand matrix*, X is called the *output matrix*, and M is called the *technology matrix*.

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$$\begin{array}{ccc} & & \text{Output} \\ & & \begin{array}{cc} C & S \\ \uparrow & \uparrow \end{array} \\ \text{Input} & \begin{array}{c} C \rightarrow \\ \\ S \rightarrow \end{array} & \left[\begin{array}{cc} \left(\begin{array}{c} \text{input from } C \\ \text{to produce \$1} \\ \text{of coal} \end{array} \right) & \left(\begin{array}{c} \text{input from } C \\ \text{to produce \$1} \\ \text{of steel} \end{array} \right) \\ \left(\begin{array}{c} \text{input from } S \\ \text{to produce \$1} \\ \text{of coal} \end{array} \right) & \left(\begin{array}{c} \text{input from } S \\ \text{to produce \$1} \\ \text{of steel} \end{array} \right) \end{array} \right] = M \end{array}$$

(C stands for the coal industry and S for the steel industry).

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Now that the individual pieces are understood, let's finish solving the problem. Let's begin by solving the matrix equation first:

$$\begin{aligned} X &= MX + D \\ X - MX &= D \\ (I - M)X &= D \\ X &= (I - M)^{-1} D \end{aligned}$$

(Note that this solution requires $I - M$ to have an inverse!)

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$$I - M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.9 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

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(I - M)^{-1} &= \frac{1}{(0.9)(0.6) - (-0.2)(-0.2)} \begin{bmatrix} 0.6 & 0.2 \\ 0.2 & 0.9 \end{bmatrix} = \begin{bmatrix} 1.2 & 0.4 \\ 0.4 & 1.8 \end{bmatrix}
\end{aligned}$$

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Now we actually work this out with the numbers from this problem

$$\begin{aligned} I - M &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.9 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \\ (I - M)^{-1} &= \frac{1}{(0.9)(0.6) - (-0.2)(-0.2)} \begin{bmatrix} 0.6 & 0.2 \\ 0.2 & 0.9 \end{bmatrix} = \begin{bmatrix} 1.2 & 0.4 \\ 0.4 & 1.8 \end{bmatrix} \\ X = (I - M)^{-1} D &= \begin{bmatrix} 1.2 & 0.4 \\ 0.4 & 1.8 \end{bmatrix} \begin{bmatrix} 20 \\ 10 \end{bmatrix} = \begin{bmatrix} 28 \\ 26 \end{bmatrix} \end{aligned}$$

So to meet the internal and final demands, \$28 billion of coal and \$26 billion of steel must be produced.

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To summarize, these are the steps to solving an input-output analysis problem:

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2. Find $I - M$.
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4. Find $X = (I - M)^{-1} D$.
5. Interpret the answer in words.

Now You Try It!

Example

The economy of a small island nation is based on two sectors, agriculture and tourism. Production of a dollar's worth of agriculture requires an input of \$0.20 *from agriculture and* \$0.15 *from tourism. Production of a dollar's worth of tourism requires an input of* \$0.40 *from agriculture and* \$0.30 *from tourism. Find the output from each sector that is needed to satisfy a final demand of* \$60 *million for agriculture and* \$80 *million for tourism.*

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Solution

\$148 *million from agriculture and* \$146 *million from tourism.*

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Example

An economy is based on three sectors: coal, oil, and transportation. Production of a dollar's worth of coal requires an input of \$0.20 *from the coal sector and* \$0.40 *from the transportation sector. Production of a dollar's worth of oil requires an input of* \$0.10 *from the oil sector and* \$0.20 *from the transportation sector. Production of a dollar's worth of transportation requires an input of* \$0.40 *from the coal sector,* \$0.20 *from the oil sector, and* \$0.20 *from the transportation sector. Find the output from each sector that is needed to satisfy a final demand of* \$30 *billion for coal,* \$10 *billion for oil, and* \$20 *billion for transportation.*

Now You Try It!

Example

An economy is based on three sectors: agriculture, manufacturing, and energy. Production of a dollar's worth of agriculture requires an input of \$0.20 *from the agriculture sector,* \$0.20 *from the manufacturing sector, and* \$0.20 *from the energy sector. Production of a dollar's worth of manufacturing requires an input of* \$0.40 *from the agriculture sector,* \$0.10 *from the manufacturing sector, and* \$0.10 *from the energy sector. Production of a dollar's worth of energy requires an input of* \$0.30 *from the agriculture sector,* \$0.10 *from the manufacturing sector, and* \$0.10 *from the energy sector. Find the output from each sector that is needed to satisfy a final demand of* \$10 *billion for agriculture,* \$15 *billion for manufacturing, and* \$20 *billion for energy.*

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An economy is based on three sectors: agriculture, manufacturing, and energy. Production of a dollar's worth of agriculture requires an input of \$0.20 *from the agriculture sector,* \$0.20 *from the manufacturing sector, and* \$0.20 *from the energy sector. Production of a dollar's worth of manufacturing requires an input of* \$0.40 *from the agriculture sector,* \$0.10 *from the manufacturing sector, and* \$0.10 *from the energy sector. Production of a dollar's worth of energy requires an input of* \$0.30 *from the agriculture sector,* \$0.10 *from the manufacturing sector, and* \$0.10 *from the energy sector. Find the output from each sector that is needed to satisfy a final demand of* \$10 *billion for agriculture,* \$15 *billion for manufacturing, and* \$20 *billion for energy.*

Solution

\$40.1 *billion from agriculture,* \$29.4 *billion from manufacturing, and* \$34.4 *billion from energy.*